Name:_______________________ Date:___________________

Ch3 Challenging 3: Sequence and Series Questions :

1. In a sequence of positive numbers, each term after the first two terms is the sum of all the previous terms. If the first term is "a", the second term is 2, and the sixth term is 56. What is the value of "a"? [Fermat #15]

2. A sequence has terms $a_1, a_2, a_3, \ldots$ The first term is $a_1 = x$ and the third term is $a_3 = y$. The terms of the sequence have the property that every term after the first is equal to 1 less than the sum of the terms immediately before and after it. That is, $n \geq 1$, $a_{n+1} = a_n + a_{n+2} - 1$. What is the sum of the first 2018 terms in the sequence. [Fermat]

3. Let t_n equal the integer closest to $\sqrt{n}$. For example, $t_1 = t_2 = 1$ since $\sqrt{1} = 1$ and $\sqrt{2} \approx 1.41$. Then $t_3 = 2$ since $\sqrt{3} = 1.73$. What is the sum of $\frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \ldots.. + \frac{1}{t_{2008}} + \frac{1}{t_{2009}} + \frac{1}{t_{2010}}$? [Fermat 2010]

4. Three distinct integers "a", "b", and "c" satisfy the following three conditions

   i) $abc = 17955$

   ii) "a", "b", and "c" form an arithmetic sequence in that order

   iii) $(3a+b),\ (3b+c),\ and\ (3c+a)$ form a geometric sequence in that order.

   What is the value of $a+b+c$? [Fermat #22]

5. A sequence $t_1, t_2, ...t_n$ is defined as follows: $t_1 = 14$ and $t_k = 24 - 5 \times t_{k-1}$ for each $k \geq 2$.
   For every positive integer "n", t_n can be expressed as $t_n = p \times q^n + r$, where "p", "q", and "r" are constants. Find the value of $p+q+r$. [Fermat #23]

6. The first term in a sequence of numbers is $t_1 = 5$. Succeeding terms are defined by the statement $t_n - t_{n-1} = 2n + 3$ for $n \geq 2$. What is the value of t_{50} ? [Fermat #24]

24. The first term in a sequence of numbers is $t_1 = 5$. Succeeding terms are defined by the statement $t_n - t_{n-1} = 2n + 3$ for $n \geq 2$. The value of t_{50} is

(A) 2700 **(B)** 2702 **(C)** 2698 **(D)** 2704 **(E)** 2706

Part C: Each correct answer is worth 8.

21. A sequence has terms $a_1, a_2, a_3, \ldots$. The first term is $a_1 = x$ and the third term is $a_3 = y$. The terms of the sequence have the property that every term after the first term is equal to 1 less than the sum of the terms immediately before and after it. That is, when $n \geq 1$, $a_{n+1} = a_n + a_{n+2} - 1$. The sum of the first 2018 terms in the sequence is

(A) $-x - 2y + 2023$ **(B)** $3x - 2y + 2017$ **(C)** y
(D) $x + y - 1$ **(E)** $2x + y + 2015$

23. A sequence $t_1, t_2, \ldots, t_n, \ldots$ is defined as follows:

$$t_1 = 14$$
$$t_k = 24 - 5t_{k-1}, \text{ for each } k \geq 2.$$

For every positive integer n, t_n can be expressed as $t_n = p \cdot q^n + r$, where p, q and r are constants. The value of $p + q + r$ is

(A) -5 **(B)** -3 **(C)** 3 **(D)** 17 **(E)** 31